Mathematical Science 2012

"Doing scientific research is quite an amazing feeling. You feel like an explorer, like an artist trying to find patterns in places where people previously thought there was chaos. So doing science is like a big adventure. It is a lot of fun and is also very important for the world."

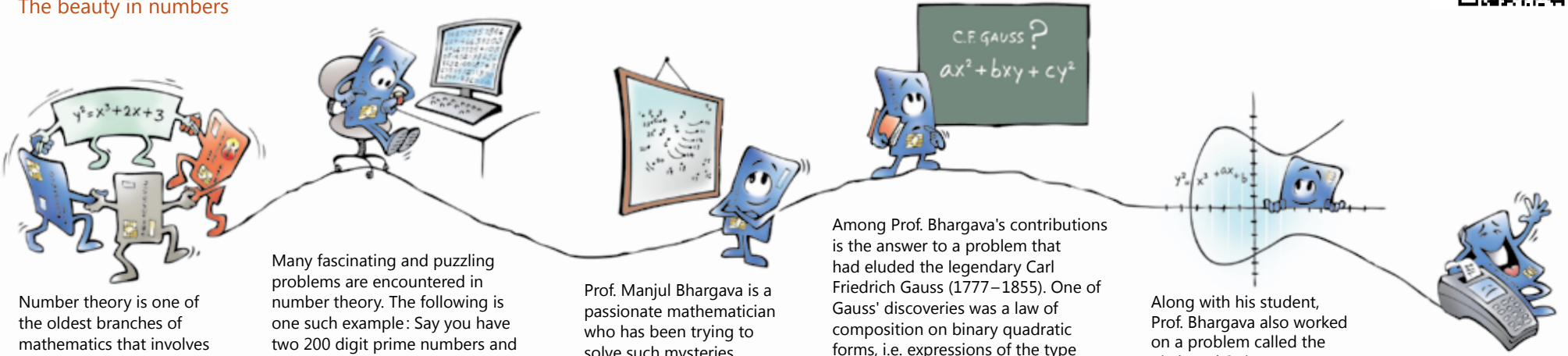
Manjul Bhargava

Professor of Mathematics, Princeton University

- A.B. in Mathematics from Harvard University
- Ph.D. in Mathematics from Princeton University

Prof. Manjul Bhargava has proved a series of beautiful theorems that greatly enhance our understanding of number fields and algebraic curves, two of the most studied topics in number theory. In particular, he showed how to count quartic and quintic number fields, and proved that the average rank of elliptic curves over the rational numbers is less than 1.

The beauty in numbers



Number theory is one of the oldest branches of mathematics that involves the study of whole numbers and how they relate to one another. It deals with finding patterns in whole numbers such as prime numbers and squares. Number theory helps in developing theories that allow one to find whole number solutions to equations such as $y^2 = x^3 + 2x + 3$.

Many fascinating and puzzling problems are encountered in number theory. The following is one such example: Say you have two 200 digit prime numbers and you feed these numbers into a computer which multiplies these numbers and gives a 400 digit number. If this 400 digit number is then fed into a computer in order to work backwards to arrive at the original two 200 digit numbers, it would take billions of years for even the most advanced computer to come up with the solution. However, no one knows why this is so. Therefore, this property is used in security related measures such as encryption schemes on the internet.

Prof. Manjul Bhargava is a passionate mathematician who has been trying to solve such mysteries. He thinks of mathematics as art. Prof. Bhargava's research in number theory deals specifically with the development of novel techniques to count objects that were previously considered inaccessible.

Among Prof. Bhargava's contributions is the answer to a problem that had eluded the legendary Carl Friedrich Gauss (1777–1855). One of Gauss' discoveries was a law of composition on binary quadratic forms, i.e. expressions of the type $ax^2 + bxy + cy^2$, with a, b and c being whole numbers that are fixed, and x and y being the variables. It was an open question as to whether this was isolated or part of a bigger theory. Prof. Bhargava showed that quadratic forms were not the only forms with such composition, but that other forms such as cubic forms also have such composition. He was also able to show that the Gauss composition is in fact only one of at least 14 such laws.

Along with his student, Prof. Bhargava also worked on a problem called the Birch and Swinnerton-Dyer conjecture, which is basically a problem in advanced calculus but which has deep implications in number theory as well. The conjecture involved elliptic curves or equations of the form $y^2 = x^3 + ax + b$.

The real world implications of the problems that Prof. Bhargava works on are not immediately evident. However, they have profound implications in areas such as encryption schemes, credit card security and internet applications.