## Mathematical Sciences 2013



"Research has one very nice aspect to it, which is that you wake up in the morning, you can think about questions that interest you the most. The difficult side is that often you don't make very much progress on it. So it is psychologically sometimes difficult. But I find it is a great career."

## Rahul Vijay Pandharipande

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- A.B. in Mathematics from Princeton University
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Prof. Rahul Pandharipande is a leader in the field of algebraic geometry. During the last 15 years, he has made profound contributions to the Gromov-Witten theory. This theory introduced in the 1990s has forged deep connections between many areas of mathematics including algebraic geometry, symplectic geometry, representation theory, etc. He excels in doing explicit computations and in finding beautiful formulae and rich structures within these theories.





In mathematics, an invariant is a quantity which remains unchanged under certain classes of transformations. Invariants are extremely useful for classifying mathematical objects because they usually reflect the intrinsic properties of the object being studied.

A circle can be drawn on a sheet of paper and is a geometric object. But a circle of radius r can be described as those points (x,y) on the plane that satisfy the quadratic equation  $x^2 + y^2 = r^2$ . Similarly, polynomial equations are algebraic objects but their solutions represent geometric objects. So, a curve in a plane is described by one equation but in space, a surface is defined by solutions of one equation and a curve is defined by solutions of two simultaneous equations.



As the degree of the polynomials that define the equations increase, the curve or the surface tends to be more twisted. For example x + y = 1is a straight line while  $x^2 + y^2 = 1$ is a circle. In mathematics, often spaces of dimensions larger than three have to be studied. For a seven dimensional space, four equations define a surface of three dimensions. Each equation reduces the dimension by one. The geometry of the resultant object has to be understood in terms of the algebraic structure of the equations that define it. Two surfaces that are presented differently can be very similar.

Invariants help in identifying similar objects or even the same object that appears in a different disquise. A circle is determined by its radius. An ellipse by its major and minor axes. There could be more than one set of invariants for the same set of objects. A triangle can be determined by either the lengths of its three sides or by one side and the three angles that add up to 180 degrees. Trigonometry provides a relation between the two sets of data.

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The objects in Prof. Rahul Pandharipande's work are Calabi-Yau '3-folds' that are geometric shapes of keen interest in mathematics and theoretical physics. They are determined by algebraic equations, typically of degree equal to the number of variables. There are two sets of invariants associated with them. The Gromov-Witten invariants 'count' the number of different maps of standard curves of varying types into a Calabi-Yau '3-fold'. The Donaldson-Thomas invariants are intrinsically defined through an algebraic construction on the same "3-fold" and provide a different set of 'counts'. The MNOP conjecture of Maulik-Nekrasov-Okounkov-Pandharipande predicted a deep and unexpected relation between the two, providing in particular an explicit recipe for computing one set from the other.

Recently, Prof. Pandharipande and his student Aaron Pixton proved this conjecture. The results are important because of their connection to string theory which is supposed be the

theory of everything!

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Uncovering the relationships between invariants