

## Towers of infinite possibility

In 1900, the International Congress of Mathematicians was held at the Sorbonne University in Paris. At the conference, David Hilbert, a German mathematician, presented 10 unsolved (at that time) problems in mathematics. Hilbert's hope was that significant progress would be made toward solving these problems by future generations of mathematicians. In 1902, Hilbert's full set of 23 unsolved problems was published (in English translation) in the *Bulletin of the American Mathematical Society* by the American mathematician, Mary Frances Winston Newson. These 23 problems set the course for much of mathematical research undertaken in the 20th century and beyond. Over the next few decades, mathematicians from around the world proceeded to work on Hilbert's problems.

Prof. Mahesh Kakde is a number theorist who works with algebraic number theory. Prof. Kakde's work makes important contributions to Hilbert's 12th problem.

Number theory has several conjectures, which as the name suggests are statements with no proofs yet. Among the conjectures that Prof. Mahesh Kakde has worked on, he is particularly known for his deep study and contributions to the Iwasawa main conjecture. Modern algebraic number theory has many conjectures relating analytic objects, namely L-functions, with arithmetic objects such as ideal class groups. Iwasawa theory, first formulated by the Japanese mathematician Kenkichi Iwasawa in the 1950s, has played a crucial

role in the development of modern algebraic number theory. It is a systematic tool for proving conjectures relating L-functions to arithmetic objects. A precise relation between the two is codified in Iwasawa main conjecture. Generalizing the work of Andrew Wiles from the 1980s, Prof. Kakde proved the non-commutative Iwasawa theory main conjecture for totally real number fields.

In the 1970s, Harold Stark and Armand Brumer posited the existence of certain numbers whose logarithms are related to values of L-functions. These special numbers, known as Brumer-Stark units, also turned out to have applications to explicit class field theory or Hilbert's 12th problem. In the 1980s, Benedict Gross used p-adic numbers as a way to obtain more information about Brumer-Stark units. In 2006, Samit Dasgupta refined the work of Gross and conjectured a p-adic formula for the Brumer-Stark units.

In one of the most exciting developments in algebraic number theory Mahesh Kakde and Samit Dasgupta resolved these conjectures, firstly, by proving the existence of the Brumer-Stark units, and then by proving the conjectured formula of Dasgupta. Besides having significant impact on number theory, the theorems led to an effective formula for computation of Brumer-Stark units and may have real-world applications in the field of cryptography.