

The Zariski Cancellation Problem: Solving one of math's greatest problems

Algebraic geometry looks at geometric shapes arising from solutions of a system of polynomial equations. Oscar Zariski, the Russian-born American mathematician, was one of the most influential mathematicians of the twentieth century who brought rigor in classical algebraic geometry and laid the foundation of modern algebraic geometry.

In 1949, Zariski posed a version of what came to be called the Zariski Cancellation Problem. This fundamental problem in mathematics asks if a geometric object has the structure of an affine space after adding a dimension to it, is then the geometric object similar to an affine space?

During the 1970s as an outcome of research of several mathematicians including S.S. Abhyankar, P. Eakin, W.J. Heinzer, M. Miyanishi, T. Sugie, T. Fujita, and P. Russell, the two-dimensional case was successfully resolved affirmatively. After that it had remained an open problem in higher dimension. It acquired a formidable reputation as one of the most difficult problems on affine spaces.

From the 1980s several mathematicians tried to solve the higher dimensional cases. In 2014, Prof. Neena Gupta published a breakthrough proof that showed that for 3-dimensional spaces adding a dimension destroys information and that the structure does not remain the same. Prof. Gupta used an example constructed by the Japanese mathematician Teruo Asanuma to prove this. Later she extended his example in higher dimensions too.

In subsequent work with her collaborators Neena Gupta has established further striking results in commutative algebra and algebraic geometry.

Prof. Gupta's work, a seminal achievement in the study of "affine algebraic geometry", has opened up new horizons and is inspiring young researchers to look into other open conjectures and unsolved problems in mathematics.