

"The instinct that there is a kind of structure underlying is what we call instinct or intuition. That's probably the most powerful thing in the doing of mathematics. Without that I don't think a person can be a practicing mathematician really. The formal reasoning comes afterwards. The intuition tells you that this is the direction that I ought to look in. Otherwise a computer would be doing mathematics!"

Mahan Mj

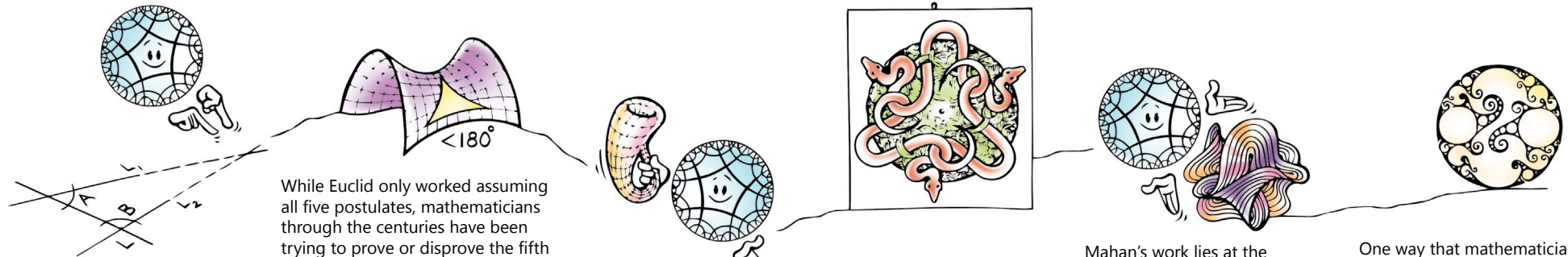
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- M.Sc. in Mathematics from Indian Institute of Technology, Kanpur
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Prof. Mahan has made a substantial impact in the fields of geometric group theory, low-dimensional topology and complex geometry. His work in all these fields is characterized by its creativity and clever use of delicate geometric arguments.



Uncovering structures beyond the observable universe



Prof. Mahan Mj works in the field of hyperbolic geometry and topology. Most of us are familiar with Euclidean geometry—the geometry of flat surfaces. When Euclid of Alexandria first formulated the principles of what is now called Euclidean geometry around 300 BCE, he came up with five postulates. Of these, the fifth postulate stated that "In a plane, through any point not on a given line only one new line can be drawn that is parallel to the original one."

While Euclid only worked assuming all five postulates, mathematicians through the centuries have been trying to prove or disprove the fifth postulate assuming only the first four. A consequence of this quest was the discovery of several non-Euclidean geometries, including hyperbolic geometry in the 19th century. Hyperbolic geometry is a geometry on a surface that is everywhere saddle-shaped. Shapes behave in peculiar and particular ways in the hyperbolic plane. In mathematical terms, while on a flat surface, the sum of angles of a triangle is always 180 degrees, in a hyperbolic plane, the sum of angles of the triangle will always be less than 180 degrees. Around the same time in the 19th century, the field of topology developed as a way of trying to understand geometry and set theory.

Topology is often called 'rubber sheet geometry'. Topologists tend to see the world in terms of stretchy, twisty objects i.e. shapes that can be twisted and glued together without tearing.

While complicated and complex, hyperbolic geometry and topology also translate into hypnotically beautiful visual patterns and shapes. Among these are fractals which are never-ending self-similar patterns which repeat infinitely. They are found everywhere around us in nature, in geometry and even in the visualization of algebraic formulae. The principles of hyperbolic geometry has also been used in art by artists like M.C. Escher in his paintings such as *Snakes*, *Circle Limit III* and *Ascending and Descending*.

Mahan's work lies at the intersection of hyperbolic geometry and topology, and specifically low-dimensional topology—the study of abstract mathematical shapes called manifolds in four or lower dimensions. Mahan was able to establish a central conjecture in a program which was established in the 1970's by the theoretical mathematician William Thurston to study hyperbolic 3-manifolds to complement his approach to his famous Geometrization Conjecture—which says that all possible 3-dimensional spaces are made up of eight types of geometric pieces.

One way that mathematicians study 3-manifolds is to study them by considering the special 'surfaces' embedded on them. A 'surface' in topology is a 2-dimensional topological manifold. Mahan's work was proof that every Kleinian surface group admits a Cannon-Thurston map (mapping in topology is a function with a special structure). He has recently extended this to include all finitely generated Kleinian groups. Mahan's work has many applications in the study of hyperbolic manifolds.