

They do it with numbers

In 2000, the Clay Mathematics Institute in the United States published a list of seven Millennium Problems. Solving any of the problems could get you a million dollars. Among these is the Riemann Hypothesis. Proposed by Bernhard Riemann in 1859, the hypothesis is a conjecture that the Riemann zeta function has its zeroes only at the negative even integers and complex numbers with real part $\frac{1}{2}$.

Prof. Ritabrata Munshi's work is related to the Riemann Hypothesis. He works on the analytic theory of L-functions. The first application of L-functions goes back to the German mathematician Johann Peter Gustav Lejeune Dirichlet in 1837. He was considering the arithmetic problem of showing many primes in arithmetic progression, such as primes of the form $5n+3$.

In order to do this, Dirichlet introduced what is called a 'function' like we use in basic calculus. A function has sometimes been described as the most important concept in mathematics. Dirichlet showed that a solution of the arithmetic problem follows from certain properties of this function. This idea was built upon by Riemann in the 1860s.

The general philosophy of an L-function is to get a 'recipe' to assign a function in calculus from a more intricate mathematical object with the hope that the more difficult problem will get transferred to a more manageable problem about the function.

In terms of representation, the properties of the L-function can be shown as an infinite rectangular strip of unit width lying on a plane. Riemann defined his famous zeta function which is the prototype of L-functions, and showed that to understand primes, one has to study this function inside the strip.

In number theory, sometimes, to solve a problem all you need to know is the size of an L-function inside the strip. In complex analysis (the branch of mathematics that investigates the functions of complex numbers), we can determine the size of a function inside the strip from knowledge we have from outside its edge.

The Riemann Hypothesis says that these functions cannot take very large values. For many applications, mathematicians just need a bound that is slightly better than the trivial or convexity bound which one gets easily from standard calculus. Improving the convexity bound is called the subconvexity problem. Prof. Munshi formulated a new approach to tackle subconvexity bound which has in turn helped tackle several other problems with which mathematicians have been grappling. Munshi's approach is based on the circle method.

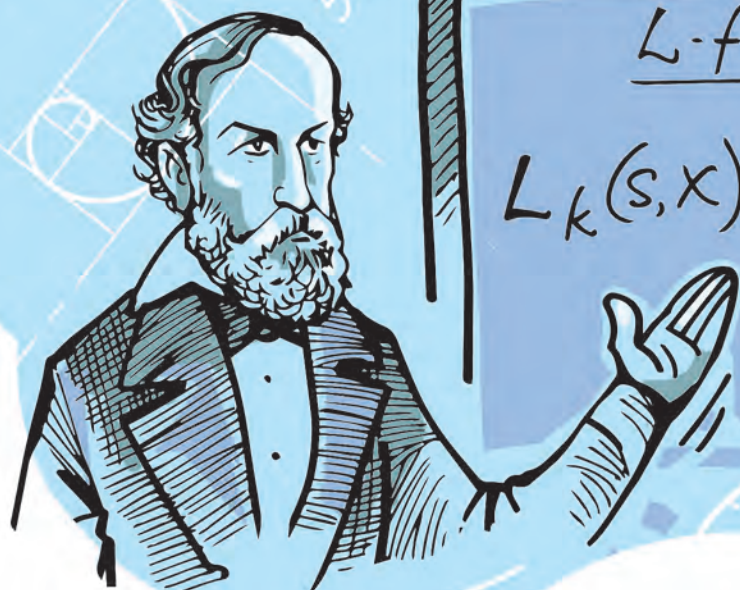
The circle method, an analytic tool which resolves arithmetic singularities, originated in work done by G. H. Hardy and Srinivasa Ramanujan in the early 1900s. The circle method is used to count integer solutions of a system of polynomial equations such as Waring's problem. Munshi's contribution to the circle method is an implementation of the level lowering trick. His most important contribution is an application of a trace formula as a form of circle method to establish subconvexity for degree three L-functions.



$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots$$

$5n+3$



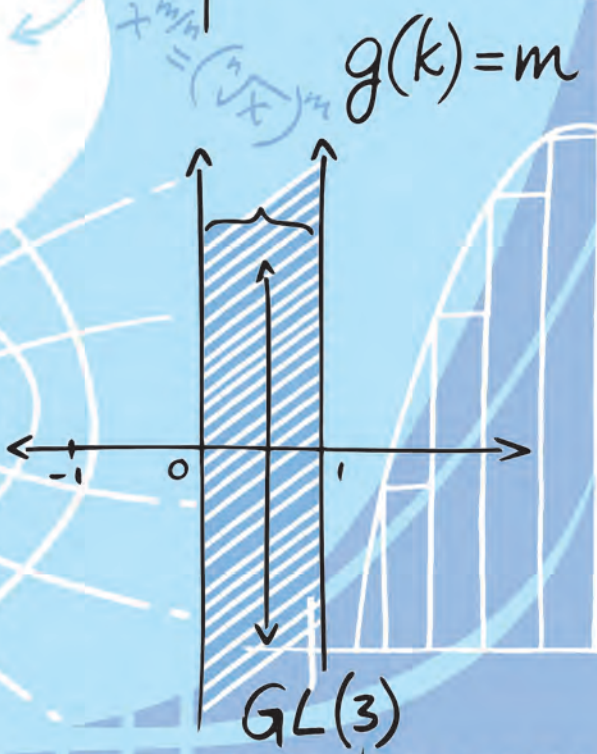
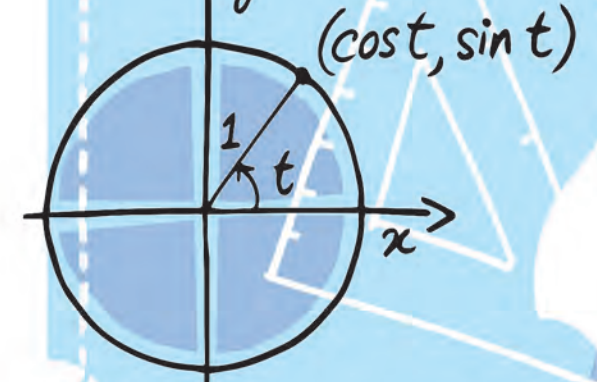
L-functions

$$L_k(s, \chi) = \sum_{n=1}^{\infty} \chi(n) n^{-s}$$

$f \pi \Sigma^+ \theta \Omega$

L-function

$\int \zeta \div \int$



$$L\left(\frac{1}{2}, \pi \otimes \chi\right) \ll M^{\frac{3}{4} - \frac{1}{308} + \epsilon}$$